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LEONHARD EULER

1707-1783

Leonhard Euler, the eighteenth-century Swiss mathematician and physicist, was one of the most brilliant and prolific scientists of all time. His work finds pervasive applications throughout physics and in many fields of engineering.

Euler's mathematical and scientific output was simply incredible. He wrote thirty-two full-length books, several of which comprise more than one volume, and hundreds upon hundreds of original articles on mathematics or science. All told, his collected scientific writings fill more than seventy volumes! Euler's genius enriched virtually every field of pure and applied mathematics, and his contributions to mathematical physics have an unending range of applications.

Euler was particularly adept at demonstrating how the general laws of mechanics, which had been formulated in the preceding century by Isaac Newton, could be applied to certain frequently-occurring types of physical situations. For example, by applying Newton's laws to the motion of fluids, Euler was able to develop the equations of hydrodynamics. Similarly, by a careful analysis of the possible motions of a rigid body, and by the application of Newton's principles, Euler was able to develop a set of equations that completely determines the motion of a rigid body. In practice, of course, material objects are not completely rigid. Euler, however, also made important contributions to the theory of elasticity, which describes how solid objects are deformed by the application of outside forces.

Euler also applied his talents to the mathematical analysis of astronomical problems, particularly the three-body problem which deals with the question of how the sun, earth, and moon move under their mutual gravitational attraction. That problem—a problem for the twenty-first century—is still not completely solved. Incidentally, Euler was the only prominent scientist of the eighteenth century who (correctly, as it turned out) supported the wave theory of light.

Euler's fertile mind often provided the starting point for mathematical discoveries that have made other men famous. For example, Joseph Louis Lagrange, the French mathematical physicist, developed a set of equations ("Lagrange's equations") which are of great theoretical importance and which can be used to solve a wide variety of problems in mechanics. The basic equation, however, was first discovered by Euler, and is usually referred to as the Euler-Lagrange equation. Another French mathematician, Jean Baptiste Fourier, is generally credited with the creation of the important mathematical technique known as Fourier analysis. Here, too, the basic equations were first discovered by Leonhard Euler, and are known as the Euler-Fourier formulas. They find wide application in many different fields of physics, including acoustics and electromagnetic theory.

In his mathematical work, Euler was particularly interested in the fields of calculus, differential equations, and infinite series. His contributions to those fields, although very important, are too technical to be described here. His contributions to the calculus of variations and to the theory of complex numbers are basic to all subsequent developments in those fields. Both topics have a wide range of applications in scientific work, in addition to their importance to pure mathematics.

Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, shows the relationship between trigonometric functions and imaginary numbers, and can be used to find the logarithms of negative numbers. It is one of the most widely used formulas in all of mathematics. Euler also wrote a textbook of analytic geometry, and made significant contributions to differential geometry and ordinary geometry.

Although Euler had a happy facility for mathematical discoveries that were capable of scientific application, he was almost equally adept in the field of pure mathematics. Unfortunately, his many contributions to the theory of numbers are too recondite to be described here. Euler was also an early worker in the field of topology, a branch of mathematics that has become very important in the twentieth century.

Last but not least, Euler made important contributions to our present system of mathematical notation. For example, he is responsible for the common use of the Greek letter π to represent the ratio of the circumference of a circle to its diameter. He also introduced many other convenient notations which are now commonly used in mathematical work.

Euler was born in 1707, in Basel, Switzerland. He was admitted to the University of Basel in 1720, when he was only thirteen years old. At first he studied theology, but he soon switched to mathematics. He received a master's degree from the University of Basel at seventeen, and when he was twenty accepted an invitation by Catherine I of Russia to join the Academy of Sciences at St. Petersburg. At age twenty-three he became professor of physics there, and at twenty-six he succeeded the famous mathematician Daniel Bernoulli in the Chair of Mathematics. Two years later he lost the sight of one eye; nevertheless, he continued to work with great intensity, turning out a long succession of brilliant articles.

In 1741, Frederick the Great of Prussia lured Euler away from Russia and induced him to join the Academy of Sciences in Berlin. He remained in Berlin for twenty-five years, returning to

Russia in 1766. Shortly afterward, he lost the sight of his other eye. Even this calamity, however, did not halt his research. Euler possessed a spectacular facility for mental arithmetic, and until the year he died (1783, in St. Petersburg, at the age of seventy-six), he continued to turn out first-rate papers in mathematics. Euler was married twice and had thirteen children, eight of whom died as infants.

All of Euler's discoveries would eventually have been made, even had he himself never lived. I think, though, that the proper criterion to apply in such a case is to ask the question: how different would science and the modern world be if the discoveries that he made had never been made at all? In the case of Leonhard Euler the answer seems fairly clear: modern science and technology would be greatly retarded, indeed almost unthinkable, without Euler's formulas, equations, and methods. A glance at the indexes of mathematics and physics textbooks shows references to: the Euler angles (rigid body motion); Euler's constant (infinite series); the Euler equations (hydrodynamics); Euler's equations of motion (dynamics of rigid bodies); Euler's formula (complex variables); the Euler numbers (infinite series); Euler's polygonal curves (differential equations); Euler's theorem on homogeneous functions (partial differential equations); Euler's transformation (infinite series); the Bernoulli-Euler law (theory of elasticity); the Euler-Fourier formulas (trigonometric series); the Euler-Lagrange equation (calculus of variations; mechanics); and the Euler-Maclaurin formula (numerical methods)—to mention only the most important examples.

In view of all this, the reader may wonder why Euler has not been ranked higher on this list. The principal reason is that although he was brilliantly successful in showing how Newton's laws could be applied, Euler never discovered any original principles of science himself. That is why such figures as Harvey, Röntgen, and Gregor Mendel, who each discovered basically new scientific phenomena or principles, have been ranked above him. Nevertheless, Euler's contributions to science, engineering, and mathematics were immense.